Radiation emission by extreme relativistic electrons and pair production by hard photons in a strong plasma wakefield

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The radiation spectrum of extreme relativistic electrons and the probability of electron-positron pair production by energetic photons in a strong plasma wakefield are derived in the framework of a semiclassical approach. It is shown that the radiation losses of a relativistic electron in the plasma wakefield scale proportionally to $\varepsilon^{2/3}$ in the quantum limit when the energy of the radiated photon becomes close to the electron energy ε. The quantum effects will play a key role in future plasma-based accelerators operating at ultrahigh electron energy.

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Crystalline fields along with laser fields are now the most important tools in laboratory strong-field physics. The electric field of atomic strings in a crystal can be up to 10^{11} V/cm near a crystallographic axis or plane [1]. When an ultrarelativistic electron with energy about 100 GeV moves along the crystallographic axis, the electric field can reach the critical electric field $E_{cr}=m^2c^3/(e\hbar)\approx 10^{16}$ V/cm in the rest frame of the electron, where e and m are the charge and mass of the electron, respectively, c is the velocity of light, and $\hbar$ is Planck's constant. At this field strength, quantum effects become significant. A number of interesting strong-field phenomena, such as quantum recoil, pair production, and spin flip, have been observed in experiments with crystals (see, e.g., [1] and references therein). Impressive progress in laser technology during recent decades promises to generate even stronger electromagnetic fields than the fields that are achievable in a crystal [2]. The nonlinear Compton effect and pair production by the nonlinear Breit-Wheeler process have been recently investigated experimentally in collisions of 49 GeV electron beam with terawatt laser pulses [3]. Quantum effects in a strong laser field are now intensively discussed [4,5].

Another example of a strong electromagnetic field that is achievable under laboratory conditions is a strong plasma field. Strong plasma fields can be generated by short intense laser pulses or by short and dense electron bunches propagating in the plasma. The plasma electrons can be completely expelled from the interaction region, leaving behind a plasma cavity ("bubble") with uniform ion density [6]. The huge space charge formed due to the electron evacuation generates a strong electromagnetic field. The strong plasma wakefield is now considered as a key element of plasma-based accelerators [7], laser-plasma x-ray radiation sources [8], and positron sources based on collisions of plasma-produced hard photons with high-Z materials [9].

A relativistic electron loaded in the plasma cavity experiences accelerating and focusing forces. The focusing force $F_\perp \approx m\omega_p^2 r/2$ is very strong an ultrarelativistic electron moving in the direction of the driver (laser pulse or electron bunch) with $p_z \gg p_\perp$, where p_z and $p_\perp$ are the longitudinal and transverse components of the electron momentum, respectively, $\omega_p=(4\pi e^2 n_0/m)^{1/2}$ is the electron plasma frequency, n_0 is the background plasma density, and r is the distance from the electron to the cavity axis. It is assumed that the axially symmetrical cavity and the driver move along the z axis. The action of the focusing force leads to transverse betatron oscillations of the electron about the z axis. The betatron frequency is $\omega_b=\omega_p(2\gamma)^{-1/2}$, where $\gamma=\varepsilon/(mc^2)$ is the Lorentz factor related to the energy of the electron, ε. The relativistic electrons undergoing betatron oscillations emit electromagnetic radiation [10]. The spectrum of the radiation becomes synchrotronlike with critical frequency $\hbar\omega_c=3\hbar\omega_p^2 r_0\gamma^2/(2c)$ when the amplitude of the betatron oscillations, r_0, is large, $p_\perp/mc\approx\omega_b r_0\gamma/c \gg 1$ [11].

The photon energy increases with increasing electron energy. When they become close to each other, the classical description of radiation emission is no longer valid. Quantum effects in strong electromagnetic fields can be characterized by the dimensionless invariants [12] $\chi=e\hbar/(m^3c^5)|F_{\mu\nu}p_\nu| \approx \gamma(F_\perp/eE_{cr})$ and $\Upsilon\approx(\hbar\omega/mc^2)(F_\perp/eE_{cr})$, where $F_{\mu\nu}$ is the field-strength tensor, p_μ is the particle four-momentum, and $\hbar\omega$ is the photon energy. χ defines the ratio of the electric field strength in the rest frame of the electron to E_{cr}. Υ determines the photon interaction with the electromagnetic field. Quantum-electrodynamic (QED) effects are important when $\chi \gtrsim 1$ or $\Upsilon \gtrsim 1$. If $\chi\gtrsim 1$ then $\hbar\omega\sim\varepsilon$ and the quantum recoil imposed on the electron by the emitted photon is strong. The invariants can be presented in the form $\chi \approx 10^{-6}\gamma$ and $\Upsilon\approx 10^{-6}(\hbar\omega/mc^2)$ for parameters $n_0 \approx 10^{19}$ cm^{-3} and $r=15$ μm, which are close to the parameters considered in Ref. [8]. The invariants are close to unity for particles with energy 500 GeV. For $n_0=10^{20}$ cm^{-3}, the threshold energy, above which quantum effects are decisive, becomes 50 GeV. While radiation generation in a plasma wakefield has been much studied in the classical limit ($\chi\ll 1$), little theory currently exists for the quantum regime ($\chi\gtrsim 1$) of radiation emission by relativistic electrons or for pair production by decay of an energetic photon in a plasma wakefield. In our paper we study these QED effects.

The motion of a relativistic electron in the plasma wakefield is semiclassical because the energy level distance of the electron in the plasma wakefield is about $\hbar\omega_b$, which is much less than ε. Yet the radiation emission can be quantum be-

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cause the energy of the emitted photon can be close to ε. In order to describe the radiation emission in a plasma wakefield, it is convenient to use the semiclassical operator method [14]. This method strongly simplifies the calculation of the radiation spectrum, since the spectrum can be expressed through the parameters of the classical trajectory of the electron. The operator method has been applied to calculation of the radiation spectrum in magnetic, Coulomb, and crystalline fields [12,14]. In the framework of the semiclassical approach, the energy radiated by an electron per frequency and per solid angle can be written as follows:

$$\frac{d^2W}{d\omega\, d\Omega_{\mathbf{n}}} = \frac{e^2\omega^2}{4\pi^2 c}\int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{+\infty} dt_2 A \exp(iB), \tag{1}$$

$$A = \frac{\varepsilon^2 + \varepsilon'^2}{\varepsilon'^2}[\boldsymbol{\beta}(t_2)\cdot\boldsymbol{\beta}(t_1) - 1] + \left(\frac{\hbar\omega mc^2}{\varepsilon'\varepsilon}\right)^2, \tag{2}$$

$$B = \frac{\varepsilon}{\varepsilon'}[\mathbf{k}\cdot\mathbf{r}(t_2) - \mathbf{k}\cdot\mathbf{r}(t_1) - \omega(t_2 - t_1)], \tag{3}$$

where $d\Omega_{\mathbf{n}}$ is the solid angle around the normal vector $\mathbf{n} = \mathbf{k}/|\mathbf{k}|$, ω and $\mathbf{k}$ are the frequency and the wave vector of the emitted photon, respectively, $\boldsymbol{\beta}(t) = \mathbf{v}(t)/c$, $\mathbf{v}(t)$ is the electron velocity, $\mathbf{r}(t)$ is the electron radius vector, and $\varepsilon' = \varepsilon - \hbar\omega$ is the electron energy after photon emission. The structure of the plasma field is taken into account through the classical trajectory of the relativistic electron defined by $\mathbf{v}(t)$ and $\mathbf{r}(t)$.

As in the classical limit [15], the main contribution to the integral in Eq. (1) comes from the neighborhood of the saddle points specified by $\dot{\mathbf{v}} \perp \mathbf{k}$. The radiation of the relativistic electron is confined within a small emission angle about $\theta_e \approx 1/\gamma$. The length l_f of the trajectory part, which gives the main contribution to the integrals in Eq. (1), is of the order of the length over which the particle is deflected by angle θ_e, $l_f \approx 2\gamma^2 c\varepsilon'/(\varepsilon\omega) \approx mc^2/F_\perp \approx 2c^2/(\omega_p^2 r)$. l_f is also called the formation length (the distance at which an electron creates a photon) [1,14].

The velocity vector of the electron undergoing betatron motion is confined within the deflection angle $\theta_d \approx \beta_\perp \approx \omega_b r_0/c$. The duration of the electron stay inside the plasma cavity, $t_s \approx L_{pc}/\Delta v$, becomes less than the betatron oscillation period $2\pi c/\omega_b$ if the electron energy is ultrahigh, $\gamma \gtrsim \gamma_{str} = 2(\pi^2\omega_p \Delta v/L_{pc})^2$, where $\Delta v = v - v_d$ is the difference in velocity between the electron and the plasma cavity, and L_{pc} is the cavity length. In this case the classical electron trajectory inside the cavity is close to a straight line. The plasma cavity velocity is equal to the driver velocity v_d (the velocity of the electron bunch or the group velocity of the laser pulse). Typically $\gamma \gg \gamma_d \gg 1$; then $t_s \approx 2\mu/\omega_p$ and $\gamma_{str} \approx \mu^2/\pi^2$, where $\gamma_d^{-2} = 1 - v_d^2/c^2$ and $\mu = (\omega_p L_{pc}/c)\gamma_d^2$. Thus, we can use the following estimate: $\theta_d \approx F_\perp t_s/(mc\gamma) \approx (\omega_p r/c)\mu/\gamma$ for $\gamma \gtrsim \gamma_{str}$. In the general case of arbitrary γ we can write $\theta_d \approx (\omega_b r/c)\min\{1, \mu(2/\gamma)^{1/2}\}$.

The radiation emission is synchrotronlike if $\theta_d \gg \theta_e$. In the opposite limit the radiation emission is dipolar. The dipole approximation is valid for the electron trajectory part, where $r \ll r_d$ and $\omega_p r_d/c \approx (2/\gamma)^{1/2}\max\{1, (\gamma/2)^{1/2}\mu^{-1}\}$. We limit our calculation to the synchrotron radiation regime since the contribution to the radiation spectrum from the trajectory part, where $r < r_d$, is negligibly small. The reason is that the energy radiated by an electron decreases as r decreases and the transverse radius of the plasma cavity, R_{pc}, is much greater than r_d because $R_{pc} \sim L_{pc} > c/\omega_p$ [16].

The used approach is valid if the focusing field does not significantly change when the electron passes the formation length along its trajectory [14]. Since the focusing force depends only on r [17], the validity condition related to the field uniformity can be written as $l_f\theta_d \ll r$. This coincides with the condition when dipole radiation is negligible, $r \gg r_d$. Another validity condition is that the formation length should be much less than the length of the electron trajectory in the plasma wakefield, $l_f \ll ct_s$. The condition fails only for a small part of the electron trajectory, where $r\omega_p/c \ll 1/\mu$. The energy radiated from this trajectory part is small and can be neglected because $\mu \gg 1$. The electron acceleration in the plasma wakefield can be ignored because the change in the energy of the electron due to the plasma acceleration is much smaller than the electron energy, at which quantum effects are significant, $\Delta\varepsilon \approx F_\parallel ct_s/2 \approx mc^2\mu^2/(2\gamma_d^2) \ll \varepsilon$ [18].

Integrating Eq. (1) over t_1 and t_2, the angular and spectral distributions of radiation energy can be calculated:

$$\frac{d^2W}{d\omega\, d\Omega_{\mathbf{n}}} = \frac{2e^2\omega^2\rho^2 N}{3\pi^2 c^3\gamma^4}(1+\Psi^2)\left\{K_{1/3}^2(x)\left[\frac{1}{2}\left(\frac{\hbar\omega}{\varepsilon'}\right)^2 + \Psi^2\frac{\varepsilon^2+\varepsilon'^2}{2\varepsilon'^2}\right] + (1+\Psi^2)\frac{\varepsilon^2+\varepsilon'^2}{2\varepsilon'^2}K_{2/3}^2(x)\right\}, \tag{4}$$

where $\Psi = \gamma\psi$, $x = (1/3)(\varepsilon/\varepsilon')(\omega\rho/c)\gamma^{-3}(1+\psi^2)^{3/2}$, ψ is the angle between $\mathbf{n}$ and $\mathbf{v}(t_k)$, t_k is the instant of photon emission corresponding to the saddle point $\mathbf{n}\cdot\dot{\mathbf{v}}(t_k) \approx 0$, ρ is the curvature radius of the electron trajectory at $t = t_k$, N is the number of betatron periods that the electron undergoes inside the plasma cavity, and $K_\nu(x)$ is the modified Bessel function.

The classical trajectory of the electron in the plasma wakefield can be approximated as $x = r_0\cos(\omega_b t)$ [11]. Thus, there are $2N$ saddle points that contribute to the integrals in Eq. (1). As in the classical limit [15], the curvature radius can be written in the form $\rho \approx (c/\omega_b)(\omega_b^2 r_0^2/c^2 - \theta^2\cos^2\varphi)^{-1/2}$, where $\psi = \theta\sin\varphi$, θ is the angle between $\mathbf{n}$ and the z axis, and φ is the azimuthal angle. In the classical limit $\varepsilon \approx \varepsilon'$, Eq. (4) reduces to the classical formula for the angular and spectral distributions of radiated energy [15]. If $\gamma > \gamma_{str}$ and the classical trajectory is close to a straight line, then there is one saddle point. In this case, $2N = 1$, and the curvature radius reduces to the form $\rho \approx \gamma mc^2/F_\perp \approx c^2/(\omega_b^2 r)$.

It is convenient to introduce the normalized power $dQ/d\xi = dP/d(\hbar\omega)$, where $Q = P/\varepsilon$, $\xi = \hbar\omega/\varepsilon$, $P = dW/dt \approx W/t_s$ is the radiation power, and t_s can be expressed in terms of N as $t_s \approx N2\pi/\omega_b$. Integrating Eq. (4) over the solid angle, the normalized spectrum can be calculated:

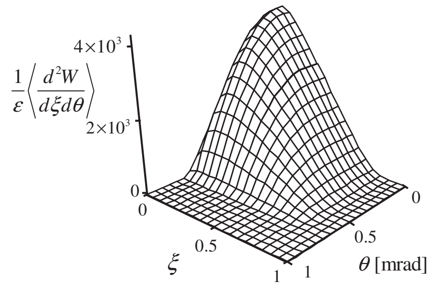

FIG. 1. Angular and frequency distribution of the normalized energy of radiation per electron, $\varepsilon^{-1}\langle d^2W/d\xi d\theta\rangle$, for an electron beam with energy 300 GeV during $t_{int}\approx 71.4\omega_p^{-1}$. The plasma bubble parameters are $R_{pc}\approx 20$ μm and $n_0=10^{19}$ cm^{-3}.

$$\frac{dQ}{d\xi}=\frac{\alpha c}{\sqrt{3}\pi\lambda_C}\frac{\xi}{\gamma}\left[\left(1-\xi+\frac{1}{1-\xi}\right)K_{2/3}(\delta)-\int_\delta^\infty K_{1/3}(s)ds\right], \tag{5}$$

where $\alpha=e^2/\hbar c$ is the fine-structure constant, $\lambda_C=\hbar/mc$ is the Compton wavelength, and $\delta=2\xi/3(1-\xi)\chi$. Equation (5) coincides with the expression for the radiation spectrum of an extreme relativistic electron in a crystalline field [see, e.g., Eq. (56) in Ref. [1]]. However, in our case χ is determined by the plasma wakefield parameters, $\chi\approx\hbar\gamma\omega_p^2 r/(2mc^3)$.

The expression for the total radiated power can be calculated by integrating Eq. (5) over the photon energy. As in the theory of electron radiation in a magnetic or crystalline field [1,12], the radiated power is $P/P_{cl}\approx 1-55\sqrt{3}\chi/16+48\chi^2$ in the classical limit, with account of quantum corrections, while in the limit $\chi\gg 1$ it can be expressed as $P/P_{cl}\approx 1.2\chi^{-4/3}$, where $P_{cl}\approx e^2\gamma^2\omega_p^4 r_0^2/12c^3$ is the classical radiated power of an electron in a plasma wakefield [11]. It follows from the obtained expressions that the electron radiation losses in the plasma wakefield scale proportionally to $\varepsilon^{2/3}$ in the quantum limit ($\chi\gg 1$), whereas the radiation losses scale proportionally to ε^2 in the classical limit.

The angular and frequency distribution of the normalized energy radiated per electron, $\varepsilon^{-1}\langle d^2W/d\xi d\theta\rangle$, during the interaction time $t_{int}=72\omega_p^{-1}$ is shown in Fig. 1 for an electron beam with energy 300 GeV. The distribution can be calculated by averaging Eq. (4) over φ and r_0. The structure of the plasma wakefield is similar to that considered in Ref. [8], where radiation emission was modeled in the classical limit. The plasma bubble parameters are $R_{pc}\approx 20$ μm and $n_0=10^{19}$ cm^{-3}. $\chi\approx 0.8$ for the electron with $r\approx R$, and the classical theory of radiation emission is no longer valid. The energy damping length of this electron because of radiation emission, $l_r\approx c\varepsilon/P$, is about five times more than l_r estimated by the classical theory. It is seen from Fig. 1 that the spectrum peaks near 40 GeV, whereas the classical theory predicts a spectrum maximum near 80 GeV. As in the classical limit [8], the radiation angle is close to the electron deflection angle $\theta_d\approx 0.7$ mrad, which is in good agreement with Fig. 1.

An electron-positron pair can be created by an energetic photon in strong electromagnetic fields [12,13]. Therefore, photon emission by relativistic electrons can cause the inverse process of pair production by a hard photon in a strong plasma field. The pair production is related to photon emission by crossing symmetry. The probability of pair production, $w_p(\Upsilon,\epsilon)$, can be calculated by the semiclassical method used above [14], where ϵ is the energy of the pair positron (electron), and $\Upsilon\approx(1/2)(\hbar\omega/mc^2)(\hbar\omega_p/mc^2)(r\omega_p/c)$ is the QED invariant. The total probability per unit time can be derived by integrating $w_p(\Upsilon,\epsilon)$ over ϵ,

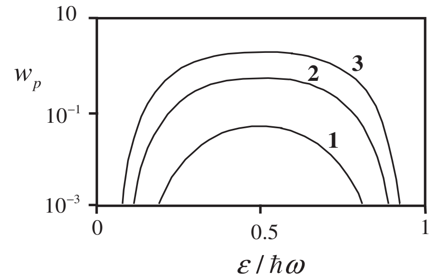

FIG. 2. Probability distribution of the normalized positron (electron) energy $\epsilon/\hbar\omega$ for the pair created by a 300 GeV photon with different values of r (1) 10, (2) 15, and (3) 20 μm, respectively. The parameters of the plasma bubble are $L_{pc}\approx 2R_{pc}\approx 40$ μm, $n_0=10^{19}$ cm^{-3}, and $\gamma_d\approx 10$.

$$W_p(\Upsilon)=\frac{C}{6\sqrt{3}\pi}\int_1^\infty\frac{8z+1}{z^{3/2}\sqrt{z-1}}K_{2/3}\left(\frac{8z}{3\Upsilon}\right)dz, \tag{6}$$

where $C=\alpha m^2c^4/(\hbar^2\omega)$. In the limit $\Upsilon\ll 1$ the probability is exponentially small, $W_p\approx 0.2C\Upsilon\exp(-8/3\Upsilon)$, while in the limit $\Upsilon\gg 1$ it is $W_p\approx 0.4C\Upsilon^{2/3}$. The pair formation length can be defined as the length it takes to deflect a created positron (electron) to an angle θ_e with respect to the photon wave vector. It is equal to l_f, which reflects the crossing symmetry of pair production and photon emission. The validity conditions of Eq. (6) are identical to those discussed above for Eq. (1).

The energy distribution $w_p(\Upsilon,\epsilon)$ of positrons (electrons) created by a photon with energy 300 GeV in a plasma bubble is shown in Fig. 2 for different values of r, where r is the distance between the photon and the z axis. The photon propagates in the direction of the z axis. The plasma bubble parameters are $L_{pc}\approx 2R_{pc}\approx 40$ μm, $n_0=10^{19}$ cm^{-3}, and $\gamma_d\approx 10$. It is seen from Fig. 2 that the distribution function peaks at $\varepsilon=\hbar\omega/2$. The positron energy spread increases as r increases. The probability of pair production by a photon crossing a bubble with $r=20$ μm is about 1.5. It is of the order of the pair production probability for a photon passing through the same distance (≈ 0.8 cm) in the field maximum of a Ge crystal cooled to 100 K [14].

In conclusion, we have calculated the radiation spectrum of energetic electrons and the probability of pair production by a hard photon in a strong plasma wakefield. Like crystals [1], a plasma can be a very efficient radiator for high-energy electrons, as well as an intense positron source. Moreover, a plasma wakefield has some advantages over crystalline and laser fields because it is a large-scale structure by comparison. The intense crystalline field is located in a very narrow layer ($\lesssim 10^{-4}$ μm) about the crystallographic axis, and the intense laser radiation varies over the laser wavelength ($\lesssim 1$ μm). The width of the high-energy electron beam is typically 10 μm and more [9], which is close to the plasma cavity size. This provides more efficient interaction of the electron beam with the strong plasma wakefield than with strong crystalline and laser fields. In addition, the ponderomotive force of the intense laser pulse pushes out the electrons from the region with strong laser field, whereas the plasma wakefield focuses the electron beam. It should be noted that the plasma wakefield can be used as a high-gradient accelerating structure [7,9], which opens the possibility for developing a compact photon and positron source.

Radiation losses in the plasma-based accelerators increase as ε^2 in the classical limit and strongly affect the dynamics of energetic electrons [18,19]. We found that the radiation losses scale proportionally to $\varepsilon^{2/3}$ in the quantum limit, which will be achievable in future plasma-based high-energy accelerators. Other quantum effects (effects accompanying collisions of charged particles [20], vacuum breakdown [4,5,21], photon splitting [5], etc.) can be important in a very strong plasma field.

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[1] U. I. Uggerhoj, Rev. Mod. Phys. **77**, 1131 (2005).
[2] T. Tajima and G. Mourou, Phys. Rev. ST Accel. Beams **5**, 031301 (2002).
[3] C. Bamber *et al.*, Phys. Rev. D **60**, 092004 (1999).
[4] G. A. Mourou, T. Tajima, and S. V. Bulanov, Rev. Mod. Phys. **78**, 309 (2006).
[5] M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006).
[6] A. Pukhov, Rep. Prog. Phys. **66**, 47 (2003).
[7] W. P. Leemans *et al.*, Nat. Phys. **2**, 696 (2006).
[8] S. Kiselev, A. Pukhov, and I. Kostyukov, Phys. Rev. Lett. **93**, 135004 (2004).
[9] D. K. Johnson *et al.*, Phys. Rev. Lett. **97**, 175003 (2006).
[10] S. Wang *et al.*, Phys. Rev. Lett. **88**, 135004 (2002).
[11] E. Esarey, B. A. Shadwick, P. Catravas, and W. P. Leemans, Phys. Rev. E **65**, 056505 (2002).
[12] V. B. Berestetskii, E. M. Lifshits, and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, New York, 1982).
[13] D. L. Burke *et al.*, Phys. Rev. Lett. **79**, 1626 (1997).
[14] V. N. Baier, V. M. Katkov, and V. M. Strakhovenko, *Electromagnetic Processes at High Energies in Oriented Single Crystals* (World Scientific, Singapore, 1998).
[15] I. Kostyukov, A. Pukhov, and S. Kiselev, Phys. Plasmas **10**, 4818 (2003).
[16] W. Lu, C. Huang, M. Zhou, W. B. Mori, and T. Katsouleas, Phys. Rev. Lett. **96**, 165002 (2006).
[17] I. Kostyukov, A. Pukhov, and S. Kiselev, Phys. Plasmas **11**, 5256 (2004).
[18] I. Yu. Kostyukov, E. N. Nerush, and A. M. Pukhov, Zh. Eksp. Teor. Fiz. **130**, 922 (2006) [JETP **103**, 800 (2006)].
[19] P. Michel, C. B. Schroeder, B. A. Shadwick, E. Esarey, and W. P. Leemans, Phys. Rev. E **74**, 026501 (2006).
[20] V. I. Berezhiani, D. D. Tskhakaya, and P. K. Shukla, Phys. Rev. A **46**, 6608 (1992).
[21] S. S. Bulanov, Phys. Rev. E **69**, 036408 (2004).